Tutorial: Quantum Information & Cold Atoms

- cold atoms as a tool in quantum information
- applications:
  - quantum computing
  - quantum communications
  - precision measurements

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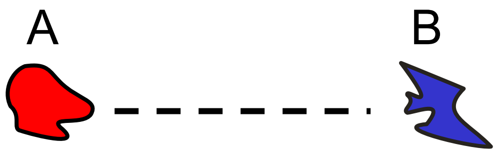
Entangled States

- entanglement



states: $|0\rangle \otimes |0\rangle$

$|1\rangle \otimes |1\rangle$ ... product states

but also ...

$$\frac{1}{\sqrt{2}}\left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right)$$ ... entangled

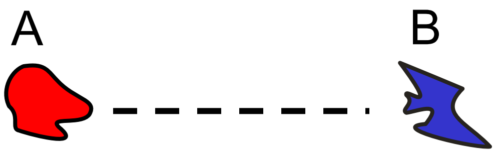
Schrödinger:
Verschränkung

- fundamental aspects of quantum mechanics
  - incompatibility of QM with LHVT
  - decoherence
  - measurement theory (?)
- applications
  - quantum communications & computing
  - precision measurement

Engineering *Entangled States*

We need ...

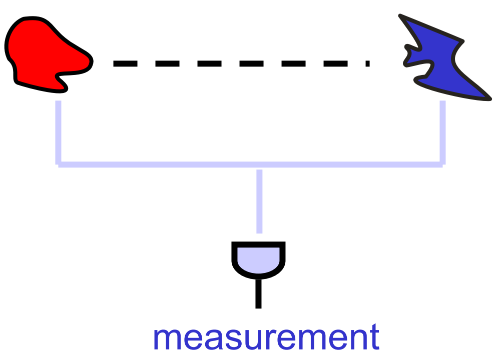
- "quantum engineering"



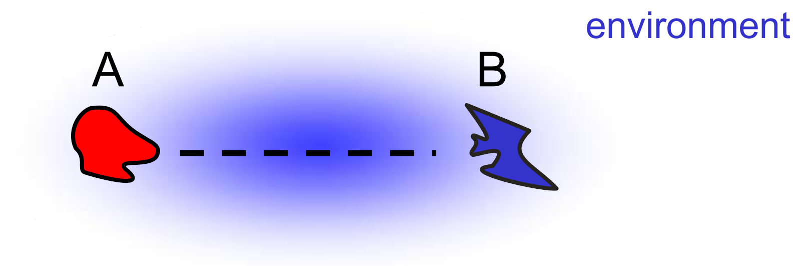
$$|a\rangle_A |b\rangle_B \rightarrow \sum c_{ab} |a\rangle_A |b\rangle_B$$

Hamiltonian evolution

- or: "quantum gambling"



- isolation



$$|\phi\rangle_A |\varphi\rangle_B |E\rangle \rightarrow |\Psi\rangle_{ABE}$$

$$\rho_{AB} = \mathrm{tr}_E |\Psi\rangle_{ABE} \langle\Psi|$$
$$\neq |\Psi\rangle_{AB} \langle\Psi|$$

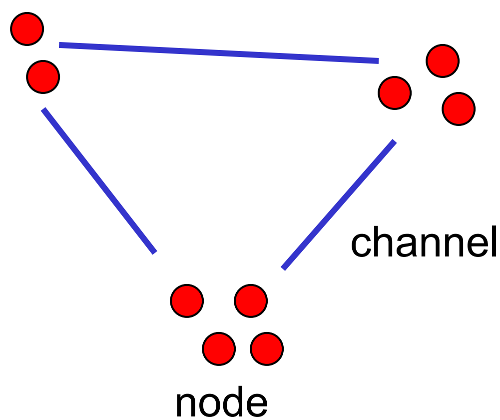
Quantum optical systems provide one of the best set-ups to create entangled states in a controlled way.

Quantum Optics Road Map

- Starting from the quantum optical tool box:
  - ✓ photons
  - ✓ atoms

  ... manipulate & couple on level of single quanta

- we assemble large systems ...

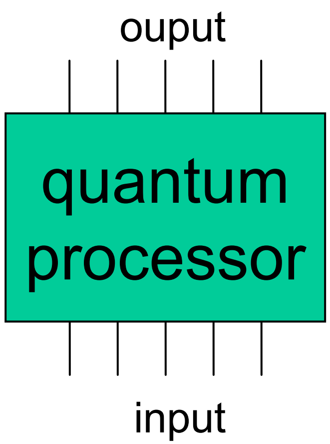


- Nodes: local quantum computing
  - store quantum information
  - local quantum processing

- Channels: quantum communication
  - transmit quantum information

Quantum information processing

- quantum computing



$\longrightarrow \quad |\psi_{\text{out}}\rangle$

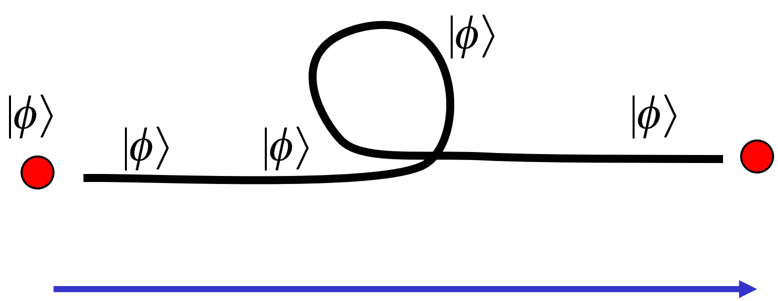
$\longrightarrow \quad |\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$

$\longrightarrow \quad |\psi_{\text{in}}\rangle$

quantum weirdness:

- ✓ superposition
- ✓ entanglement
- ✓ interference
- ✓ nonclonability and uncertainty
- ✓ no decoherence!

- quantum communications

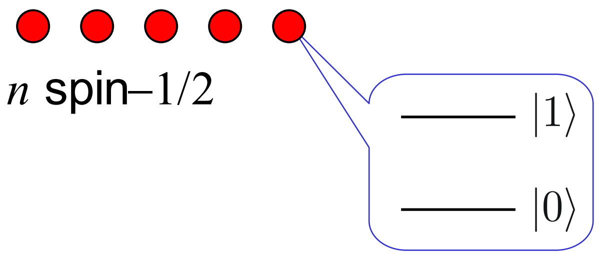


transmission of a quantum state

- ✓ teleportation
- ✓ crytography

Qubits, Quantum Gates etc.

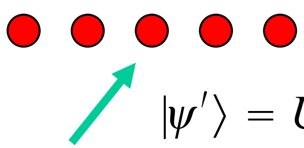
- quantum bits or qubits



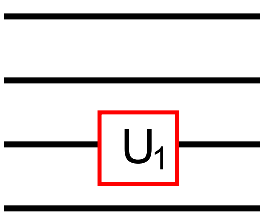
example: two qubits

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

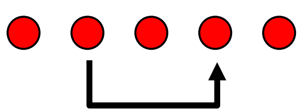
- single qubit gate

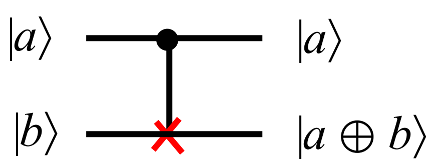


$$|\psi'\rangle = \hat{U}_1|\psi\rangle$$



- two qubit gate





Controlled-NOT

1. Cold atoms as quantum memory

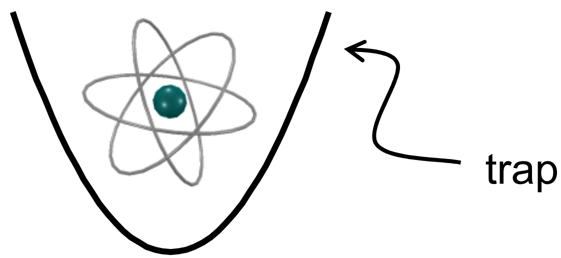
- technique; cooling and trapping of atoms
- decoherence properties

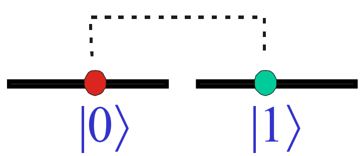
2. How to entangle atoms?

1. Cold atoms as quantum memory

- cold atoms

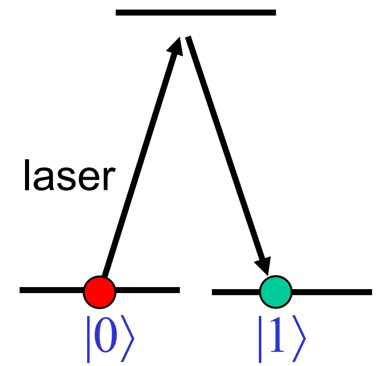
single trapped atom:



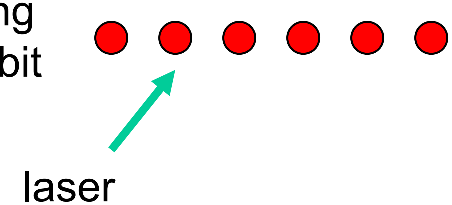


qubit in *longlived* internal states

- single qubit gates

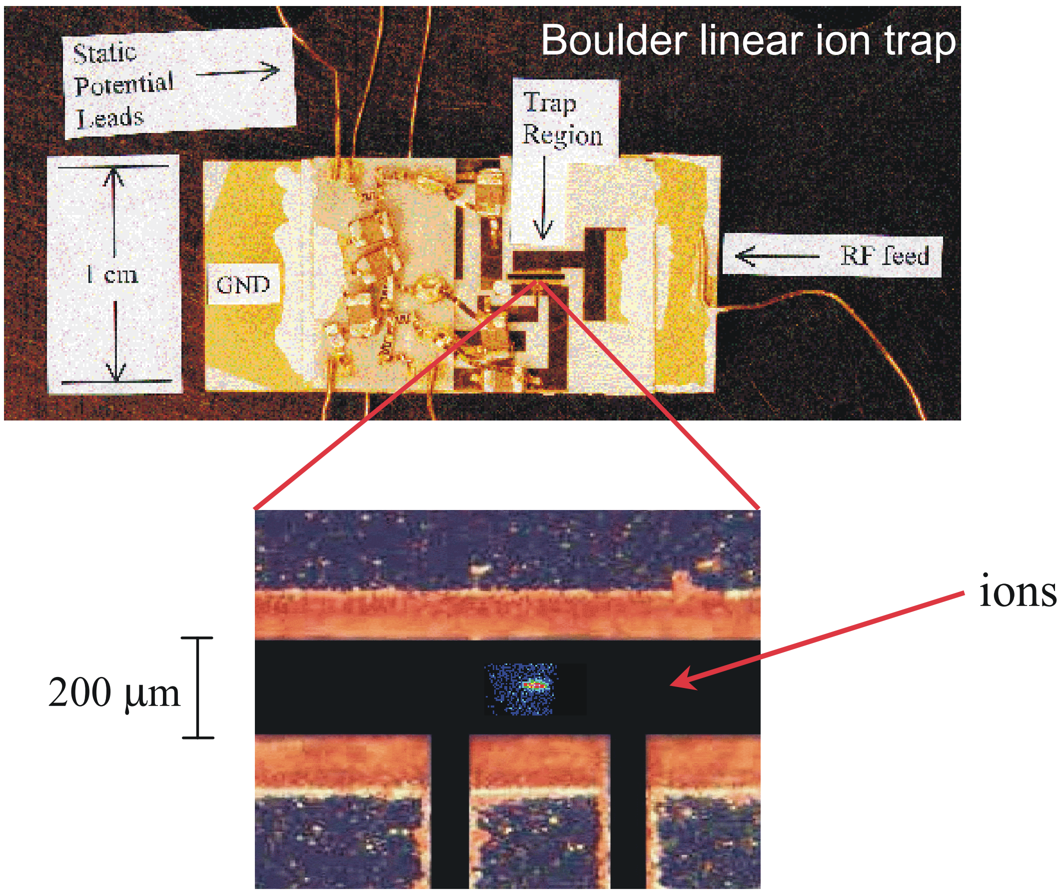


addressing single qubit



Remarks: Traps ...

- ion traps



issues:

✓ conservative potential (heating?)

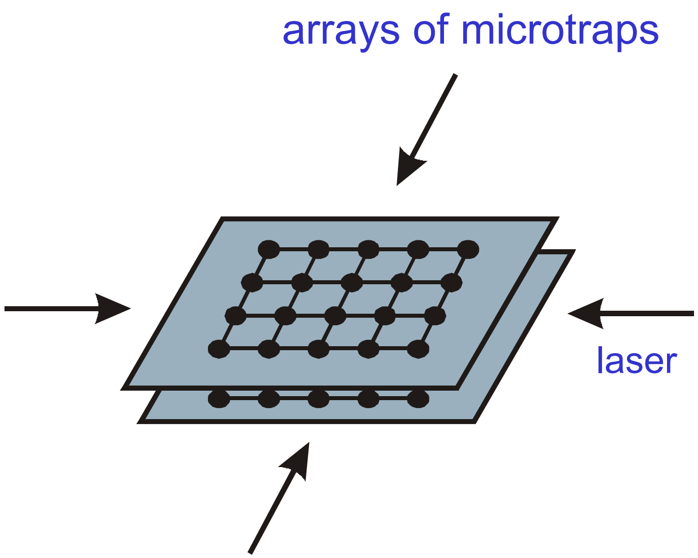
✓ single atom loading

✓ laser cooling to ground state

NIST Boulder, Innsbruck, Munich, Hamburg, Aarhus, Oxford, London, ...

Remarks: Traps ...

- far-offresonance optical lattice

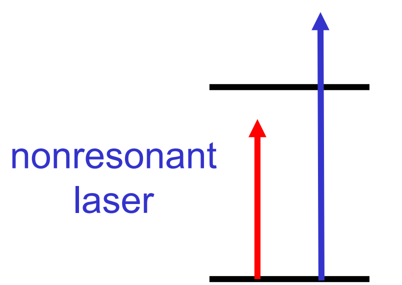


issues:

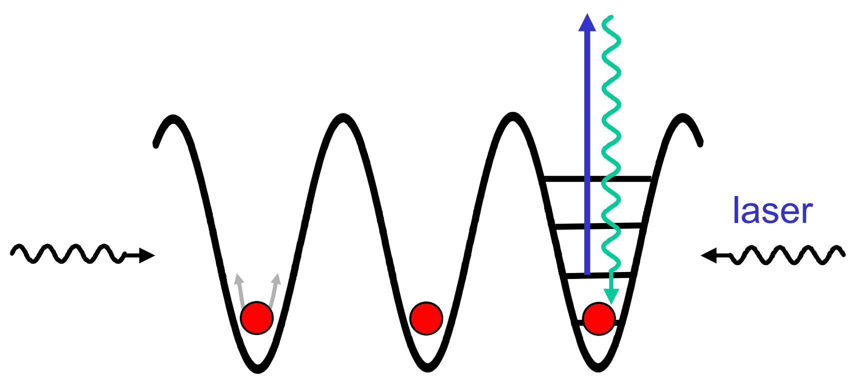
✓ conservative potential

✓ single atom loading (?!)

✓ laser cooling



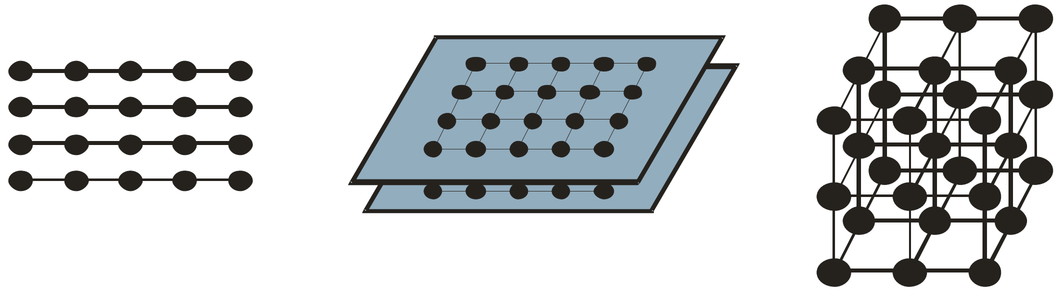
AC Stark shift



optical lattice

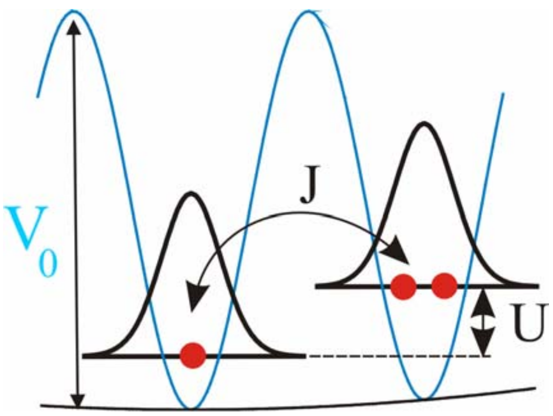
Remarks: Traps ...

- loading a lattice from a Bose Einstein condensate
  - *regular* filling with exactly 1, 2 or 3 atoms per lattice site via Mott insulator quantum phase transition (Bose Hubbard model)
  - *large* number of atoms



theory: Innsbruck 1998

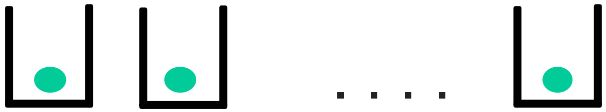
exp: Munich 2001



Bose Hubbard model

- superfluid phase

- Mott insulator



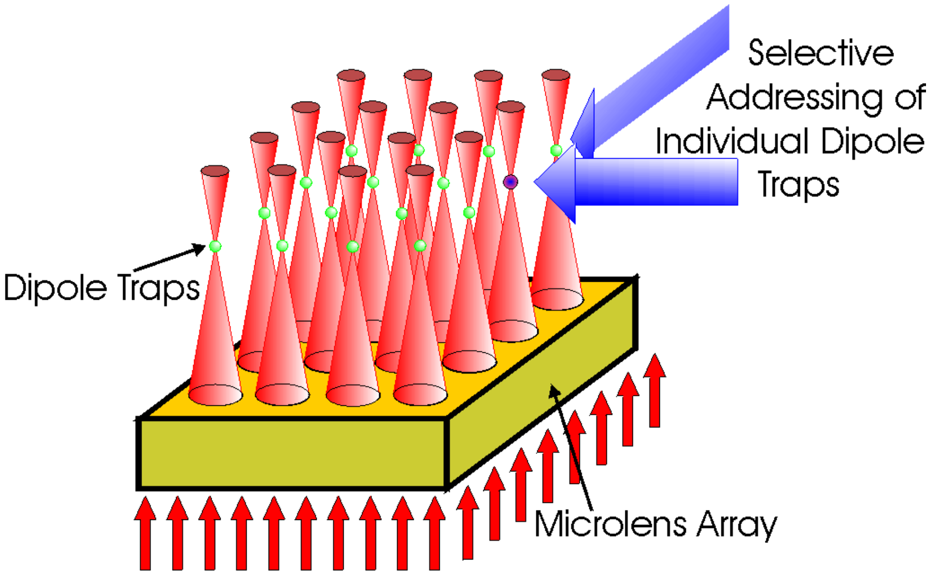
$$b_1^\dagger b_2^\dagger \ldots b_M^\dagger |\mathrm{vac}\rangle$$

"Fock states"

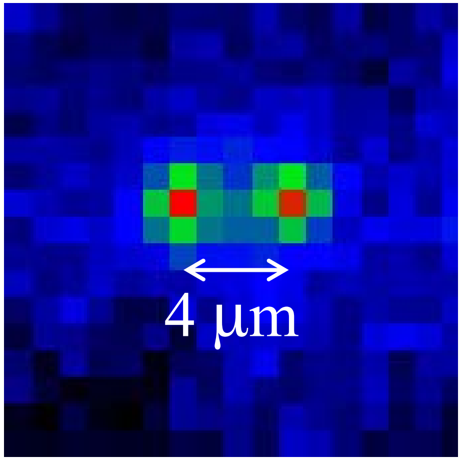
Remarks: Traps ...

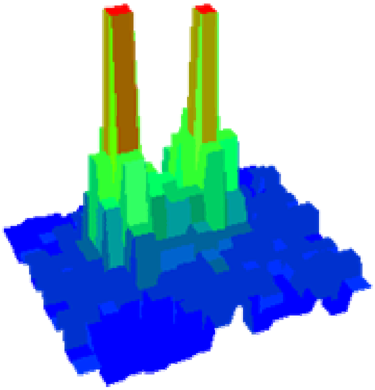
- single atom FORTs

array of FORTs (Hannover)



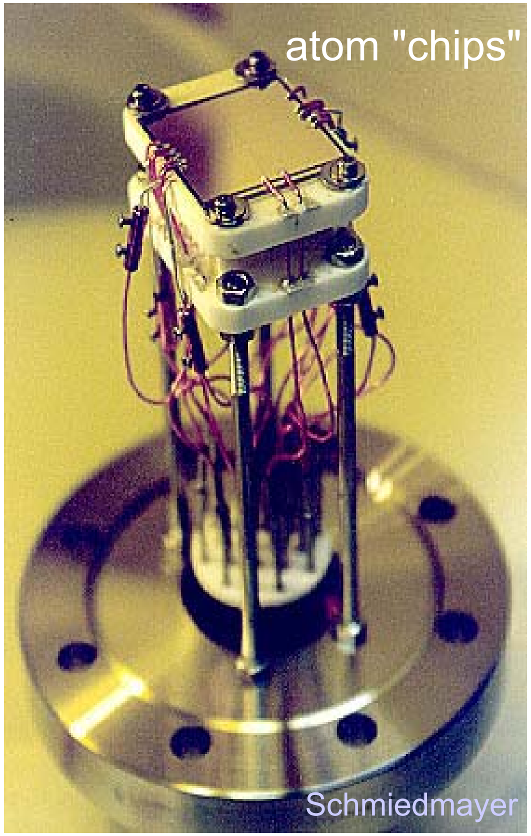
two movable single-atom FORTs (Orsay)





Remarks: Traps ...

- magnetic traps



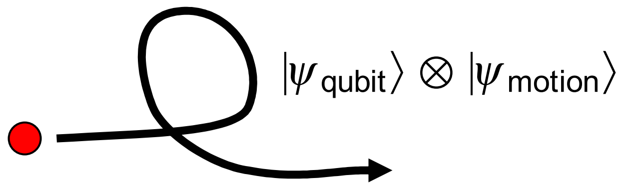
Heidelberg, Munich,
Harvard, Orsay

issues:

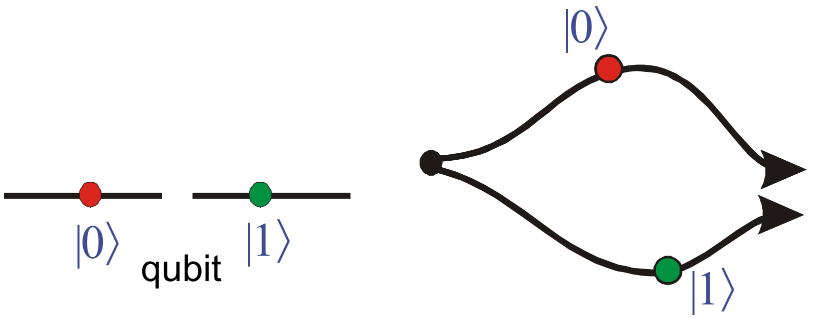
- ✓ conservative potential
  surface effects (?)
- ✓ single atom loading (?)
- ✓ laser cooling (?)

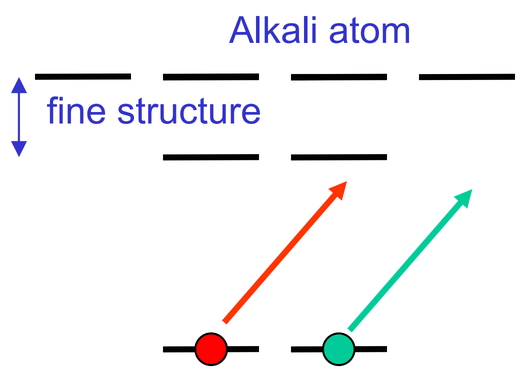
... and tricks with traps

- We can move atoms around



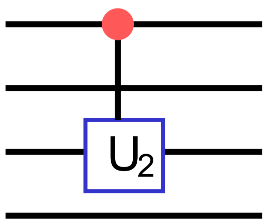
- optical traps: internal state dependent potentials





2. Engineering entanglement: two-qubit gates

- implement entanglement of two qubits



example:
phase gate

$$
\begin{aligned}
|00\rangle &\rightarrow |00\rangle \\
|01\rangle &\rightarrow |01\rangle \\
|10\rangle &\rightarrow |10\rangle \\
|11\rangle &\rightarrow e^{i\phi}|11\rangle
\end{aligned}
$$

- How?

✓ auxiliary collective mode as data bus

✓ controllable two body interactions

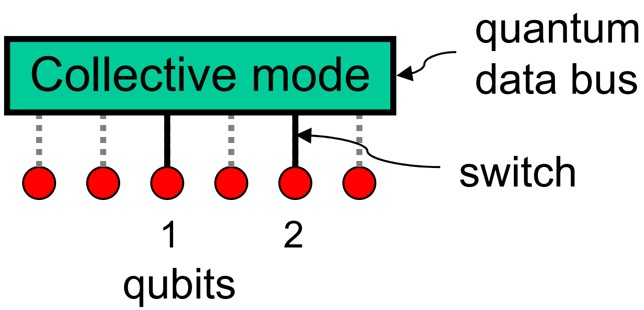
difficult

✓ dynamical phases

✓ geometric phases

Concept 1: two-qubit gates via quantum databus

- Entanglement via collective auxiliary quantum degree of freedom



Examples:

Ion traps

Cavity QED

state vector:

$$|\psi\rangle = \sum_{\{x\}} c_x \underbrace{|x_{N-1} x_{N-2} \ldots x_0\rangle}_{\text{quantum register}} \otimes \underbrace{|\text{collective mode}\rangle}_{\text{data bus}}$$

gate:

swap

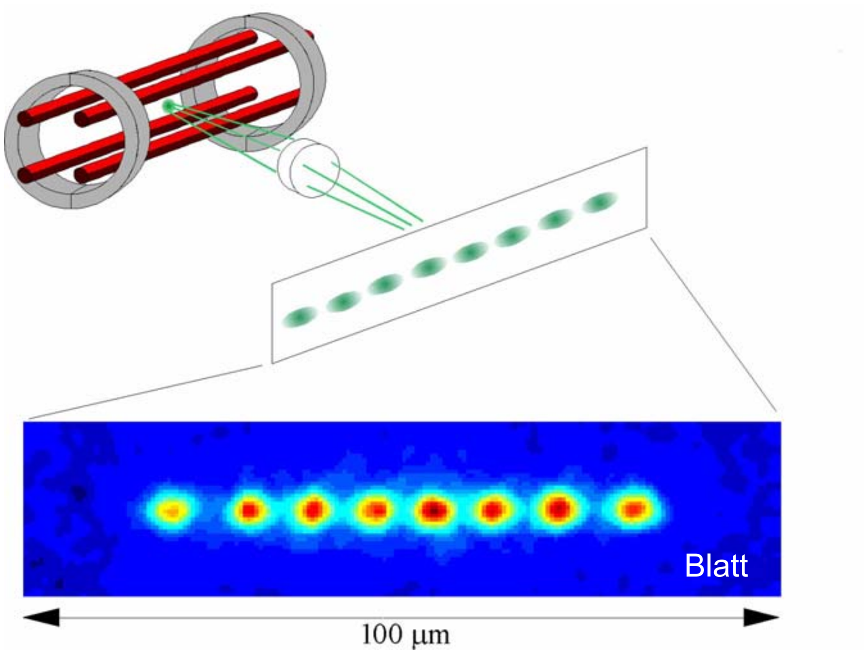
requirement: cooling of the collective mode (= prepare a pure state)

Ion Trap Quantum Computer

theory: Innsbruck, Aarhus, London, Brisbane ...

exp: NIST Boulder, Innsbruck, Munich, Oxford ..

- Cold ions in a linear trap



- Qubits: internal atomic states
- Quantum gates: entanglement via exchange of phonons of quantized center-of-mass mode

- Achievements:
  - entanglement of four ions
  - Bell measurements
  - individual addressing
  - ground state laser cooling

- State vector

$$|\Psi\rangle = \sum c_x |x_{N-1}, \ldots, x_0\rangle_{\text{atom}} \quad |0\rangle_{\text{phonon}}$$

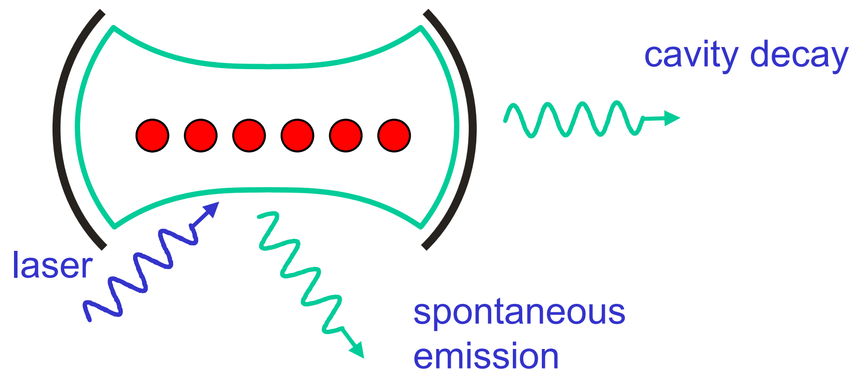
quantum register databus

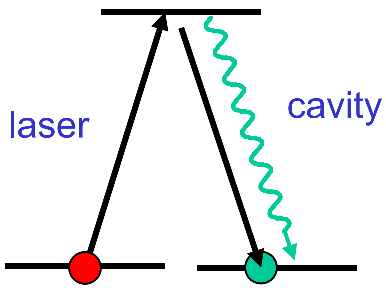
Optical Cavity QED

theory: Innsbruck, London ...

exp: Caltech, Georgia Tech, Munich

- *optical* photons in a high-Q cavity as "data bus"



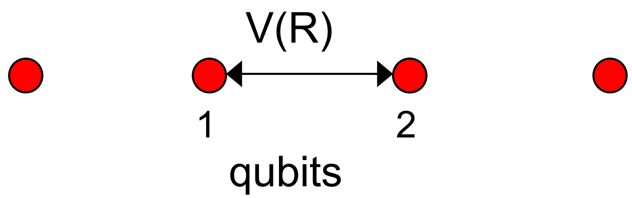


adiabatic pasage

decoherence free subspace

Concept 2: two-qubit gates via two-body interactions

- Controlled two-body interaction



We must design a Hamiltonian

$$H = \Delta E(t)\,|1\rangle_1\langle 1| \otimes |1\rangle_2\langle 1|$$

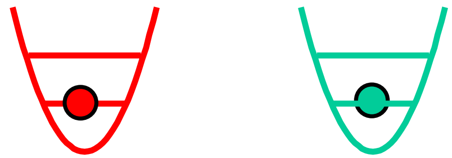
so that

$$|1\rangle_1 \otimes |1\rangle_2 \rightarrow e^{i\phi}|1\rangle_1 \otimes |1\rangle_2$$

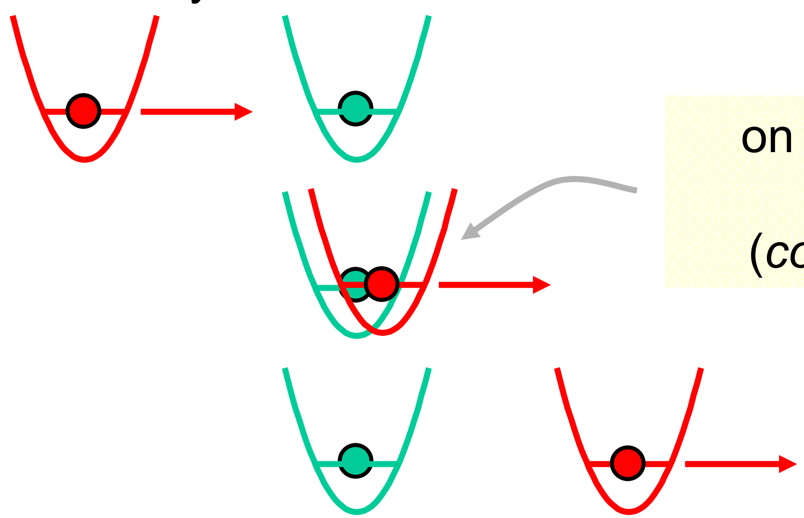
- physical mechanisms
  - ✓ optical dipole – dipole (Albuquerque)
  - ✓ cold collisions (Innsbruck)
  - ✓ Rydberg – Rydberg (Innsbruck + Harvard + Storrs)

Cold controlled collisions

- consider two atoms in different internal states stored e.g. in an optical lattice



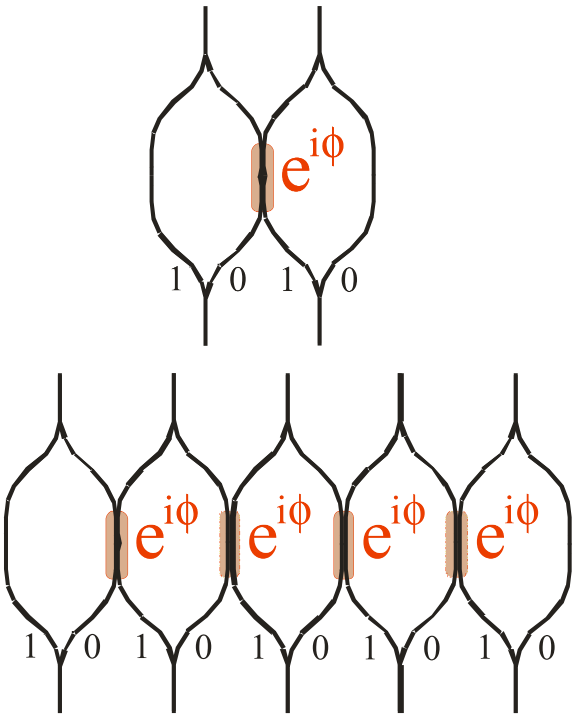
- we *move* the lattice in a state dependent way, and induce a collision: the collision is sufficiently *slow* not to excite oscillations

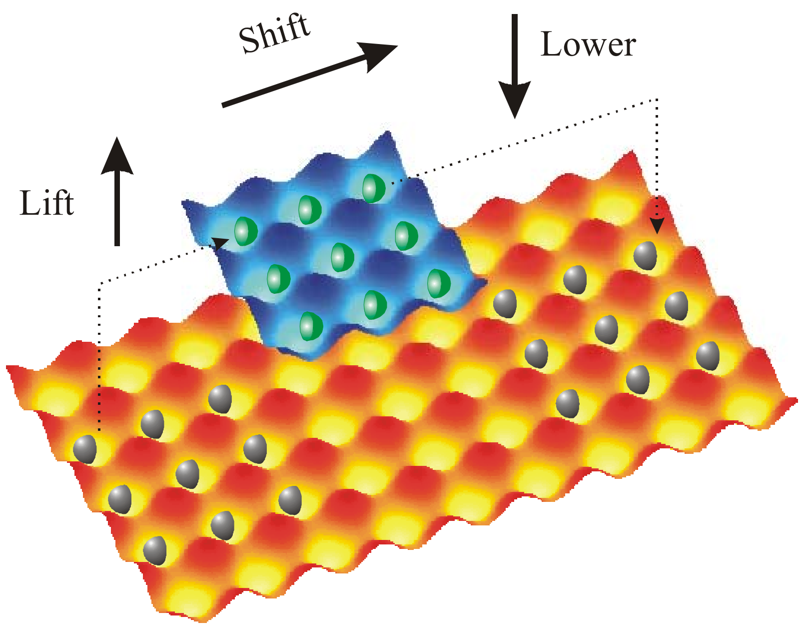


- phase gate via collisions ...

Entanglement

- parallelism



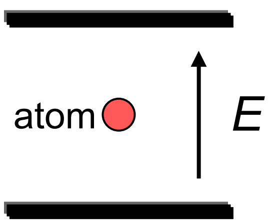


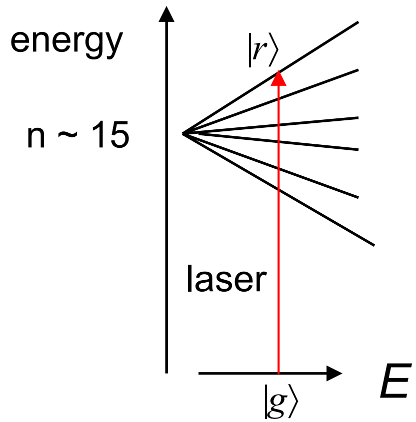
- fidelity F~99%
- ... slow ☹

Rydberg – Rydberg interactions

theory: Harvard + Innsbruck + Storrs, Aarhus

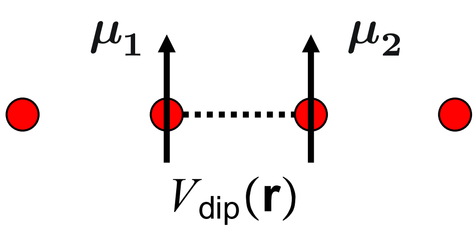
- Rydberg atom in constant electric field
  - setup
  - linear Stark effect
  - permanent dipole moment





$\mu \sim n^2$ huge!

- Large dipole-dipole interaction



$E \sim 1$kV/cm

R~ $\lambda_{\text{opt}}/2 \sim 300$ nm

$\Delta E \sim 60$ GHz

$V_{\text{dip}} \sim 4$ GHz

for $n \sim 15$

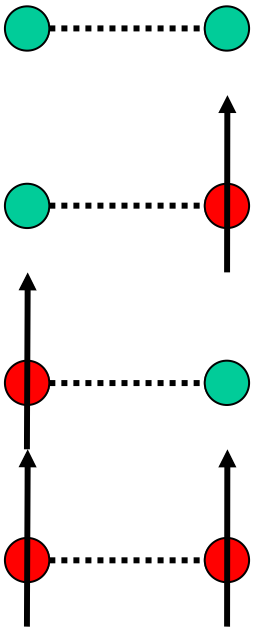
- ***2 qubit quantum gate***

$$|00\rangle \longrightarrow |00\rangle$$

$$|01\rangle \longrightarrow |01\rangle$$

$$|10\rangle \longrightarrow |10\rangle$$

$$|11\rangle \longrightarrow e^{i\phi}|11\rangle$$

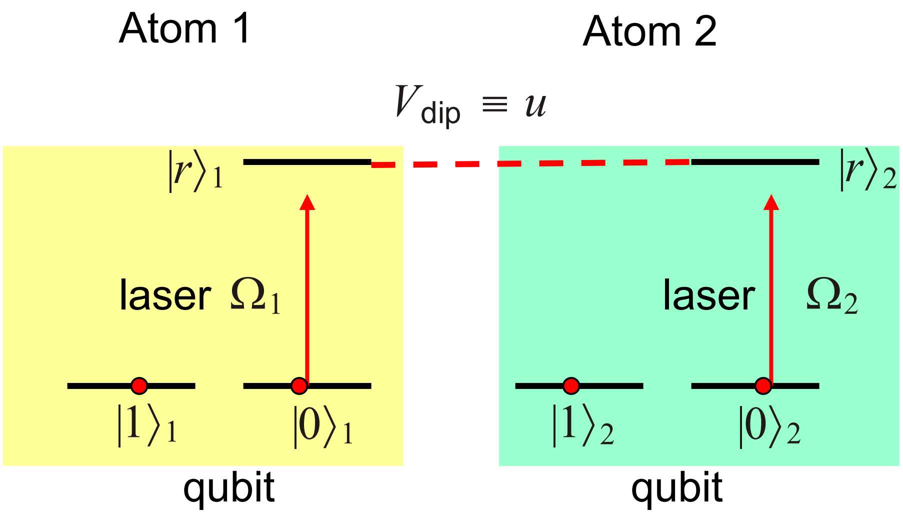


fast gate

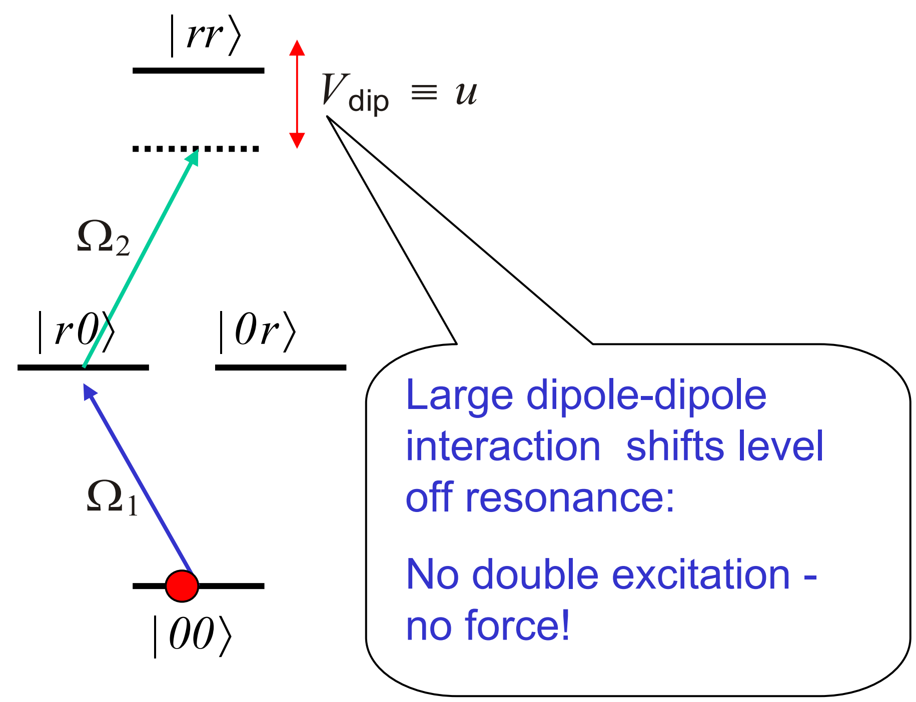
energy shift for time interval = phase

"Dipole blockade"

- atomic configuration



- dipole blockade



3. Mesoscopic atomic ensembles

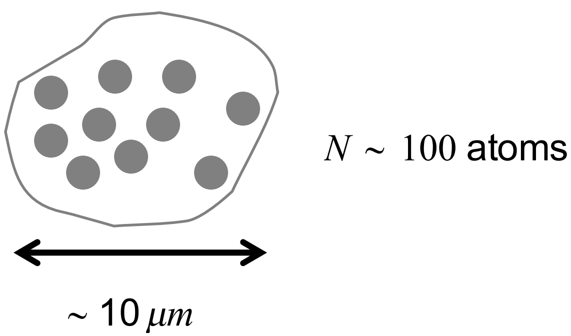
- idea
  - dipole blockade mechanism

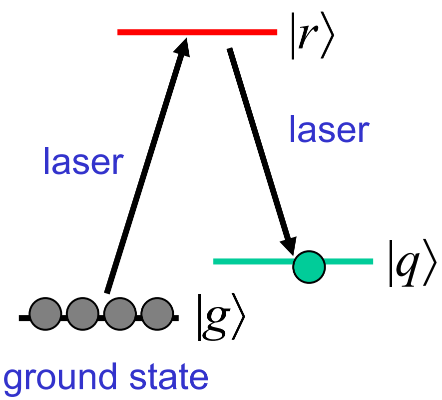
theory: Harvard + Innsbruck, exp: Storrs, ...

Configuration

- *mesoscopic* atomic ensembles (instead of microscopic quantum objects)

- coherent manipulation of *collective excitations* of atomic ensembles

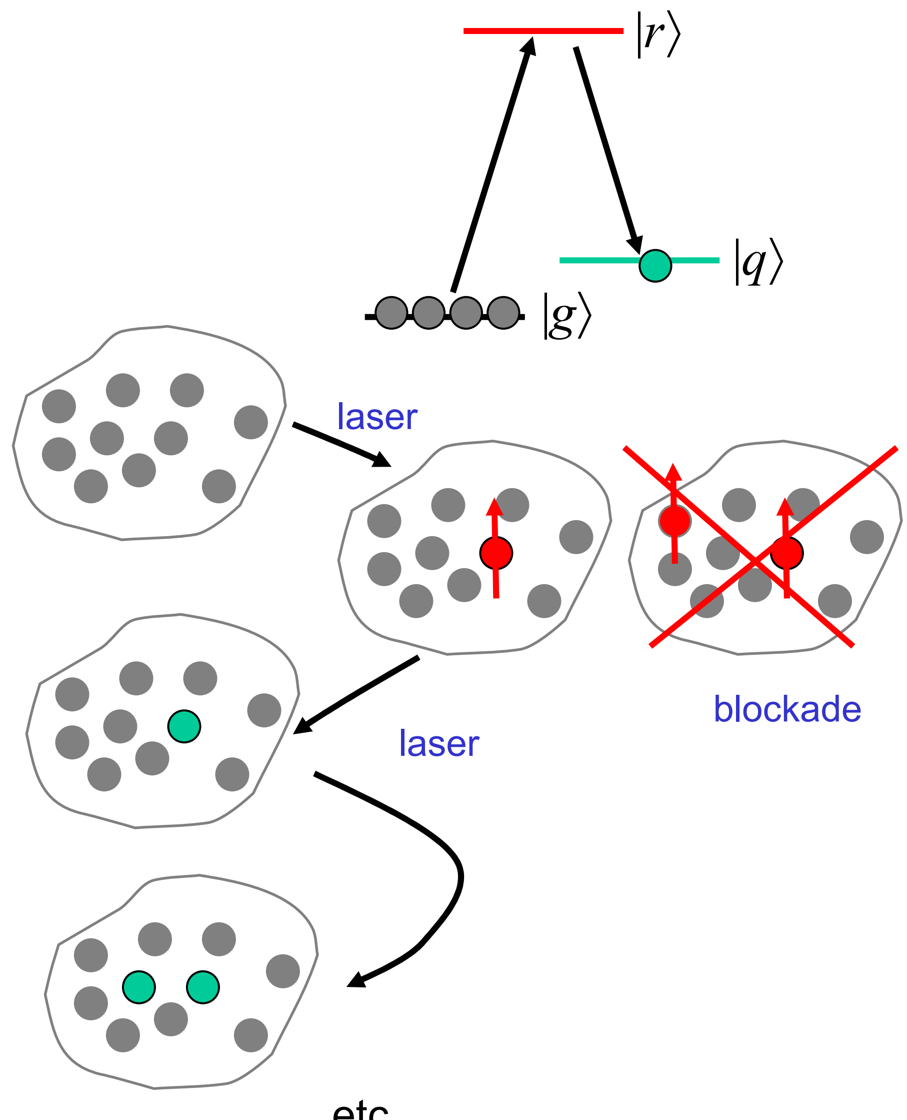




-underlying physics:

dipole blockade

Manipulating collective excitations



- ground state

$$|g^N\rangle = |g_1\rangle|g_2\rangle\ldots|g_N\rangle$$

- one excitation (Fock state)

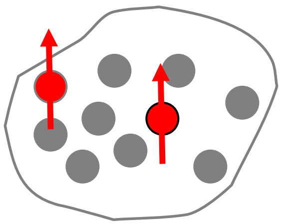
$$|g^{N-1}q\rangle \sim \sum_i |g_1\rangle\ldots|q_i\rangle\ldots|g_N\rangle$$

- two excitations

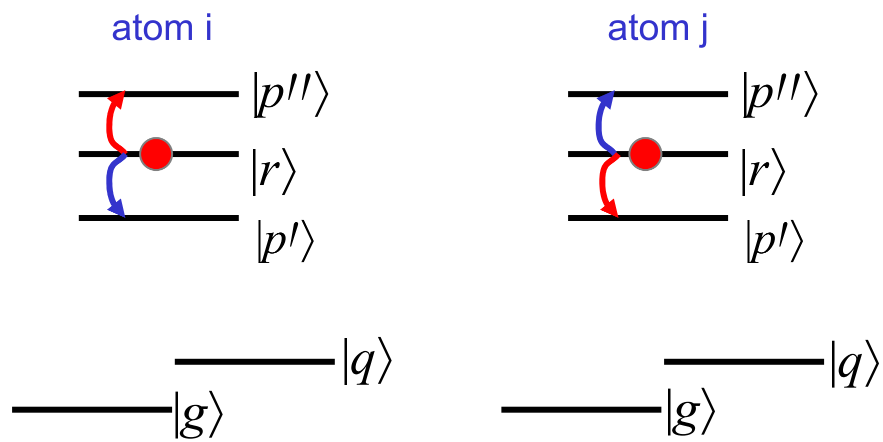
$$|g^{N-2}q\rangle \sim \sum_{i,j} |g_1\rangle\ldots|q_i\rangle\ldots|q_j\rangle\ldots|g_N\rangle$$

We can store and manipulate qubits.

- actual configuration



hopping via resonant dipole-dipole interaction



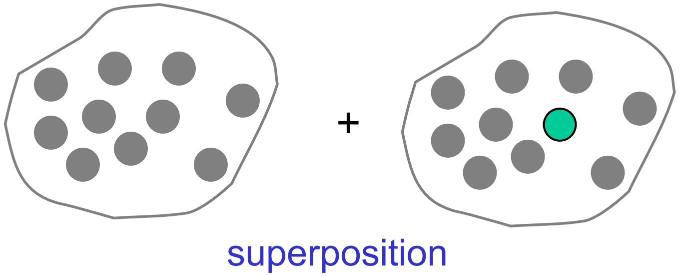
resonance condition: $E_r - E_{p'} = E_{p''} - E_r$

idea: Cote, Lukin

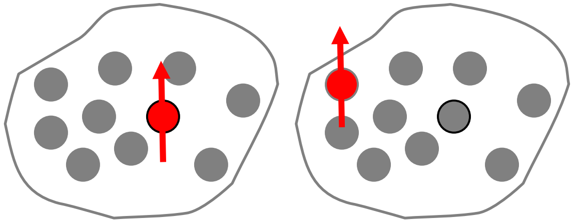
cont.

- qubits

$$|\psi\rangle = \alpha|g^N\rangle + \beta|g^{N-1}q\rangle$$



- entanglement of ensembles



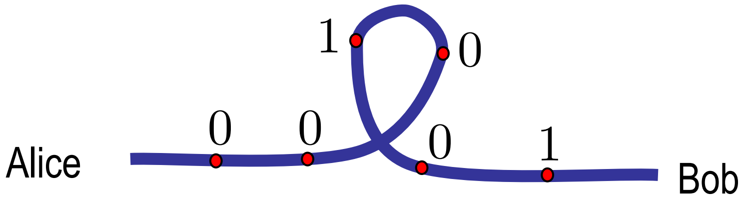
4. Quantum Communication

- Question:
  - what can we do with 10 qubit quantum computers?

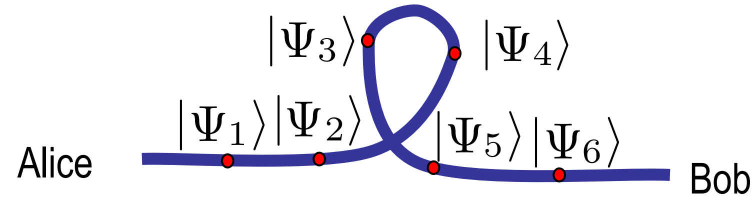
- Example:
  - quantum communication with memory and quantum error correction = quantum repeater

Quantum Communications

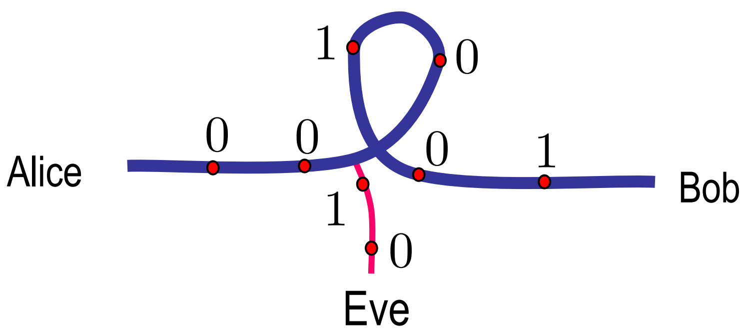
- classical communications

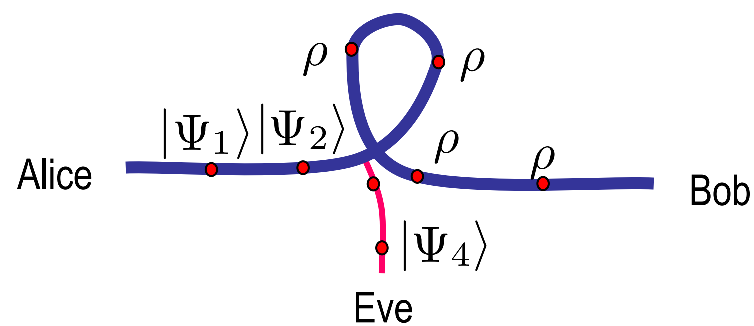


- quantum communications



✓ networks
✓ cryptopgraphy

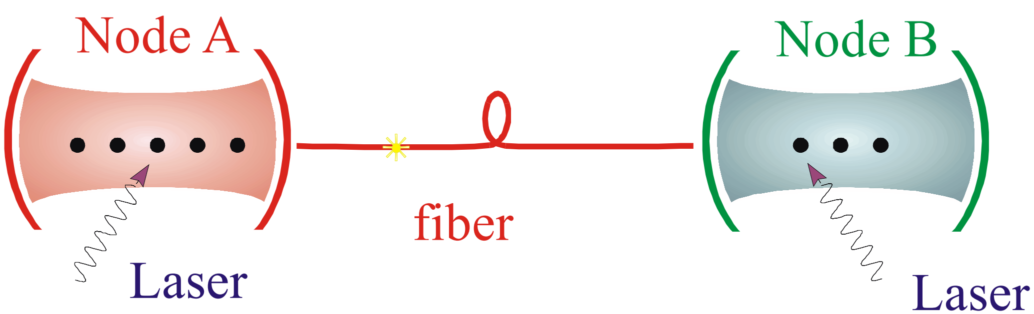




Optical Interconnects

- A cavity QED implementation

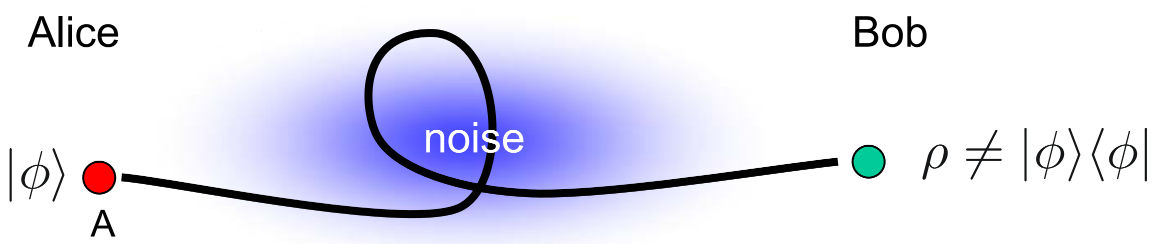
Optical cavities connected by a quantum channel



• memory: atoms → • databus: photons → • memory: atoms

Quantum Communication Protocols

- Problem: the basic problem of quantum communication is noise / decoherence



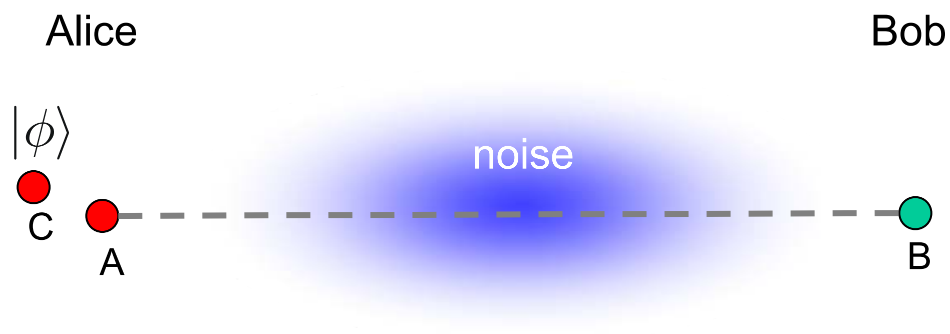
pure state $\longrightarrow$ mixed state:

fidelity $F = \langle\phi|\rho|\phi\rangle < 1$

- Solution: quantum communication in the presence of noise is based on
  - teleportation
  - purification (error correction)

1. Teleportation

Bennett et al. PRL '93; exp with photons: Innsbruck, Rome, Caltech



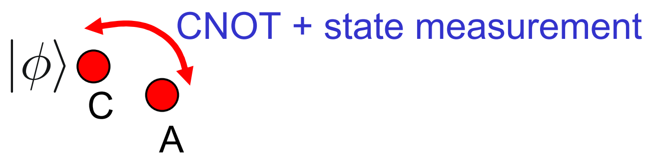
$$|\psi\rangle_{AB} \sim |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B$$ (singlet)

EPR pair

- Assume: Alice and Bob share a singlet state (given resource)
- Idea: Alice can transmit a qubit to Bob without physically sending the qubit

Teleportation protocol:

1. Alice performs a CNOT followed by a measurement of A and C



$$\begin{array}{l}\text{4 possible outcomes} \\ \forall|\phi\rangle \text{ with prob } \frac{1}{4}\end{array} = \begin{cases} 00 \\ 01 \\ 10 \\ 11 \end{cases}$$

2. Alice tells Bob the measurement outcome

3. Rotation

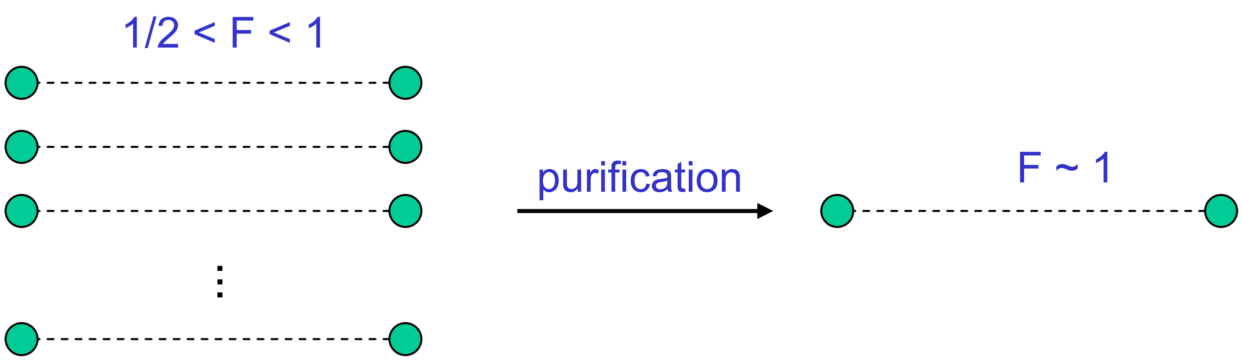
The state has been teleported to B.

How do we get the EPR pair?

2. Purifying EPR pairs

Bennett et al., Deutsch et al. PRL '95

- A noisy quantum channel allows us to generate *many noisy* EPR pairs.



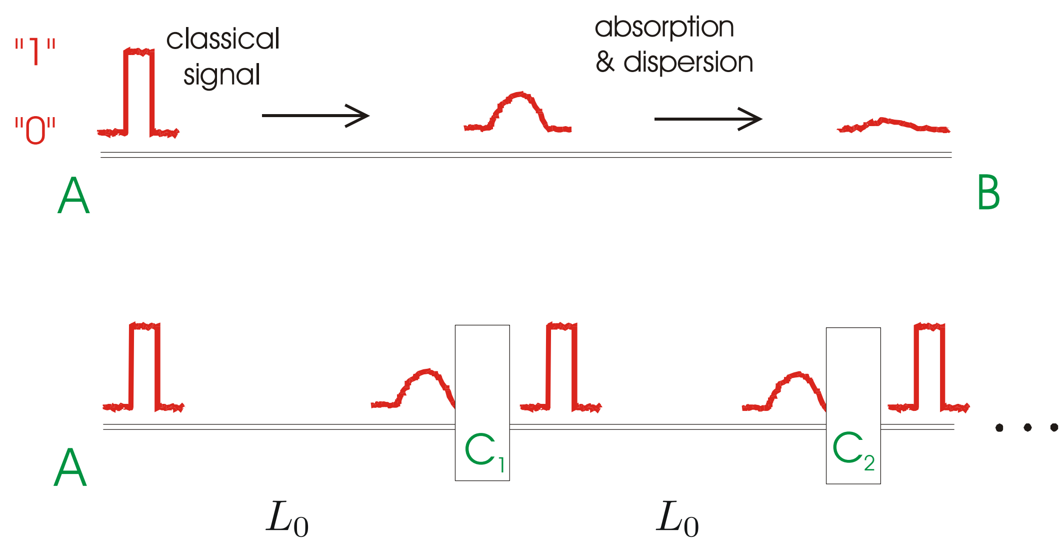
If $F > 1/2$ we can purify and obtain one EPR pair with $F \sim 1$.

To do this we need a small quantum computer.

- In summary: quantum communication via a noisy channel:
  - we generate one high fidelity EPR pair by purification
  - we teleport the quantum state

3. Quantum Repeater

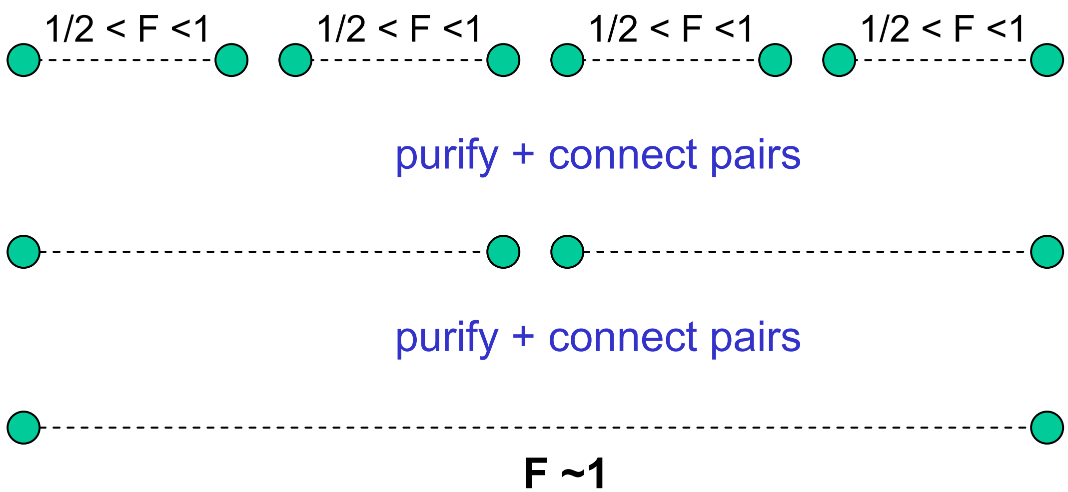
- classical repeater



- quantum repeater:
  - we cannot clone a quantum state!?
  - Idea: purify segments of EPR pairs ... and teleport H. Briegel et al. PRL '98

cont.

- Quantum repeater protocol: generate a long distance EPR pair

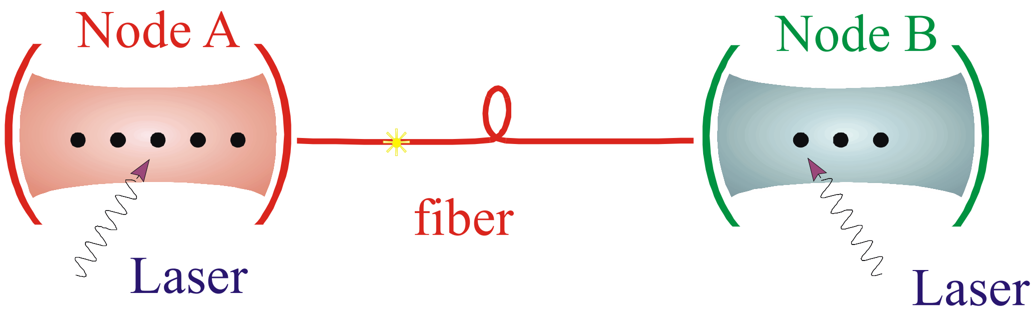


We use this EPR pair to telport the quantum state.

- required resources for a photonic channel
  - ~ 10 qubit quantum computer for 10.000 km
  - tolerate errors on the few % level

Other (easier?) implementations

- single atoms, single photons and high-Q cavities: "quantum engineering"



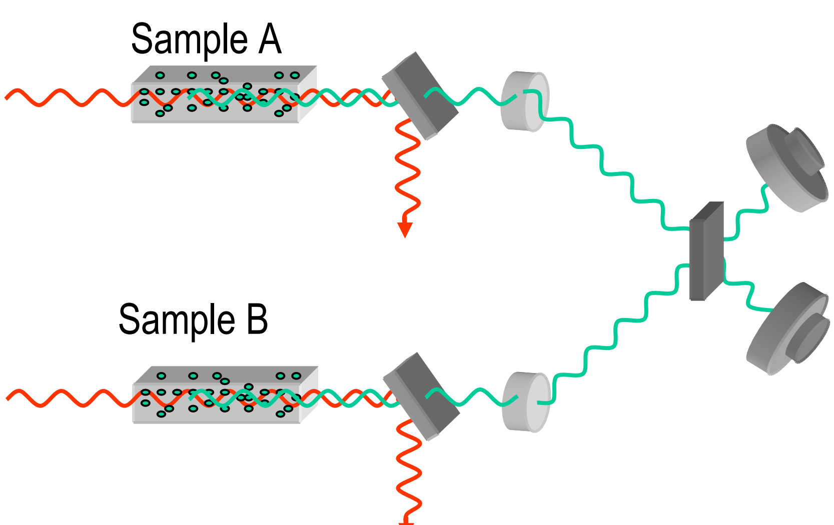
✓ deterministic

✓ strong coupling on single quantum level

- memory: atoms → - databus: photons → - memory: atoms

difficult !

- atomic ensembles, free space, or low-Q cavities: "quantum gambling"



✓probabilistic

✓scheme

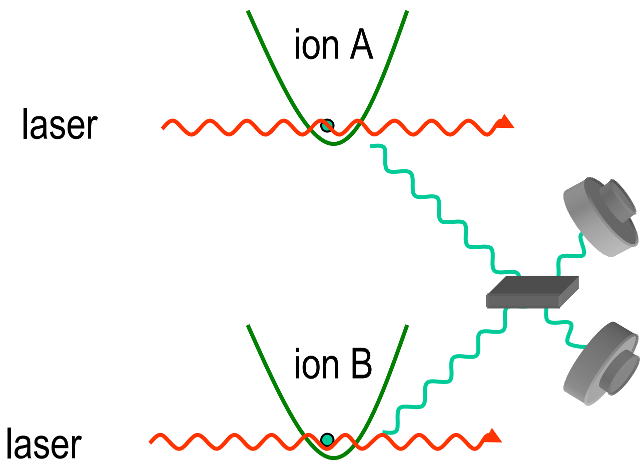
✓noise tolerant

✓linear optics

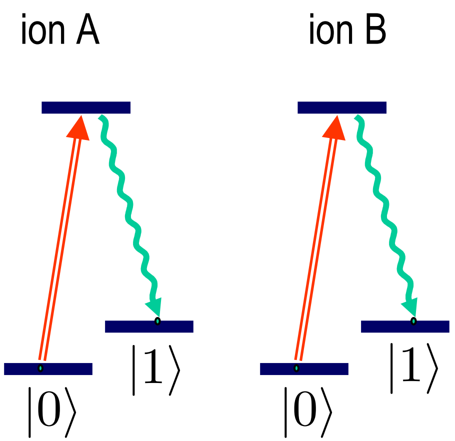
simpler !

To illustrate the ideas ...

Single Atoms



Internal states

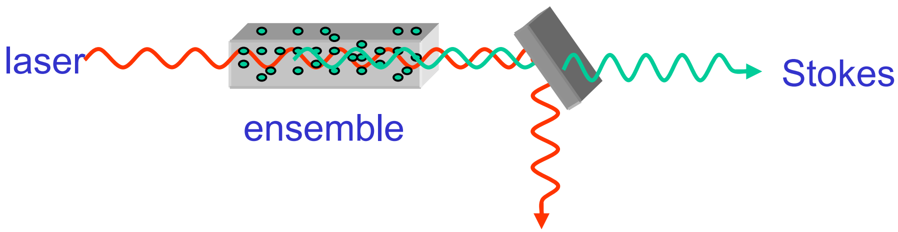


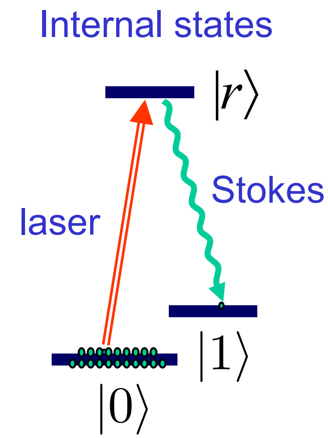
- Weak (short) laser pulse, so that the excitation probability is small.
- If no detection, pump back and start again.
- If detection, an entangled state is created.because we do not know which atom emitted the photon

$$|0,1\rangle \pm |1,0\rangle$$

Atomic Ensembles

- system: cloud of cold atoms





- Raman process:

$$|0\rangle^{\otimes N} \equiv |\text{vac}\rangle \text{ (atomic ground state)}$$

$$\rightarrow \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} |0_1 \ldots 1_i \ldots 0_{N_a}\rangle \equiv a^\dagger |\text{vac}\rangle \quad \text{(single atomic excitation)}$$

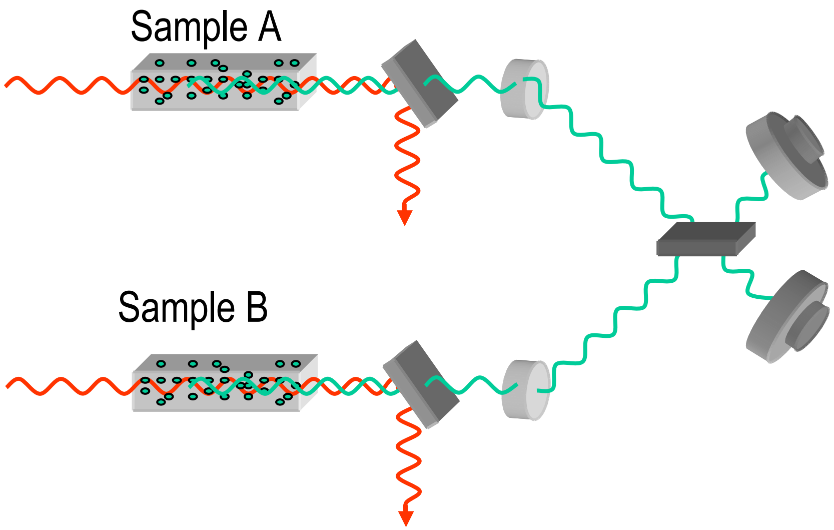
$$[a, a^\dagger] \approx 1 \quad \text{(for weak excitation)}$$

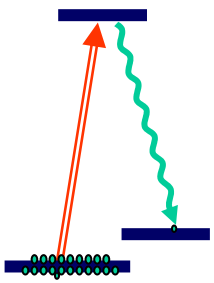
- state of atomic collective mode + Stokes photon

$$|\phi\rangle = |\text{vac}\rangle + \sqrt{p_c}\, a^\dagger c^\dagger_{\text{Stokes}} |\text{vac}\rangle + O(\sqrt{p_c}^{\,2}) \quad (p_c \ll 1$$

... analogous to parametric downconversion

Generation of Entanglement (Ensembles)





$$|\phi\rangle_A \otimes |\phi\rangle_B = \left(|\text{vac}\rangle_A + \sqrt{p_c}\, a^\dagger c_s^\dagger |\text{vac}\rangle_A\right) \otimes \left(|\text{vac}\rangle_B + \sqrt{p_c}\, b^\dagger c_s^\dagger |\text{vac}\rangle_B\right)$$

measurement gives

$$|\psi_{AB}^{\pm}\rangle = (a^\dagger \pm b^\dagger)|\text{vac}\rangle$$
$$\equiv |1_a, 0_b\rangle \pm |0_a, 1_b\rangle$$

We have generated entanglement between collective atomic states

Continuous Variable Teleportation

- Instead of qubits we consider now continuous variable quantum states

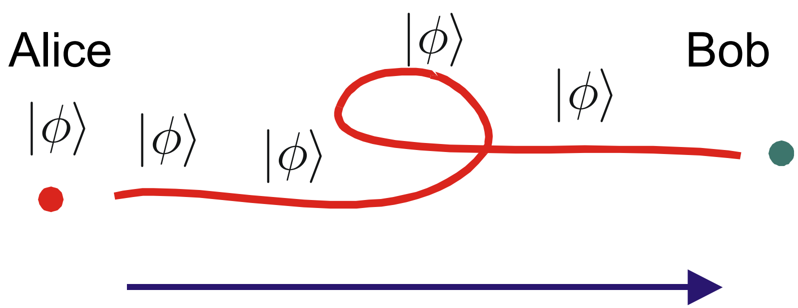
$$|\phi\rangle = \int dx\, |x\rangle\, \phi(x)$$

$\hat{x}$ … position

$\hat{p}$ … momentum

$$[\hat{x},\hat{p}] = i$$

- transmission of a cv state



- continuous variable teleportation

Vaidman
Braunstein
Kimble (exp)

$$|\text{EPR}\rangle_{AB} \sim \int dx\, |x\rangle_A |x\rangle_B$$

$$\sim \int dp\, |p\rangle_A |-p\rangle_B$$

$$(\hat{x}_A - \hat{x}_B)|\text{EPR}\rangle = x_1 |\text{EPR}\rangle$$

$$(\hat{p}_A + \hat{p}_B)|\text{EPR}\rangle = p_1 |\text{EPR}\rangle$$

Teleportation with Squeezed Light

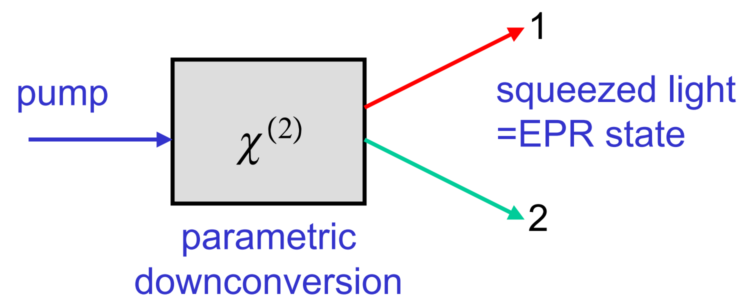
S. Braunstein, H.J. Kimble et al., PRL '98; Science '99

- Two-mode squeezed light:

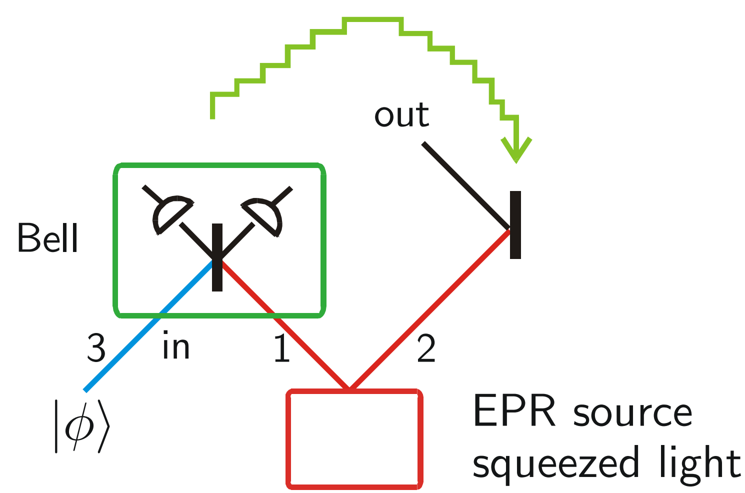
electric field $E^{(+)} \sim a\, e^{ikx-i\omega t}$

$$a = \hat{x} + i\hat{p}$$

quadrature components

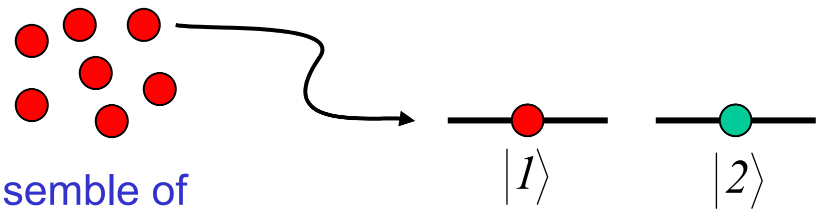


- Scheme



Atomic ensembles as quantum memory for cont var states

- We consider an ensemble of N atoms



ensemble of atoms

two-level atom =spin -1/2

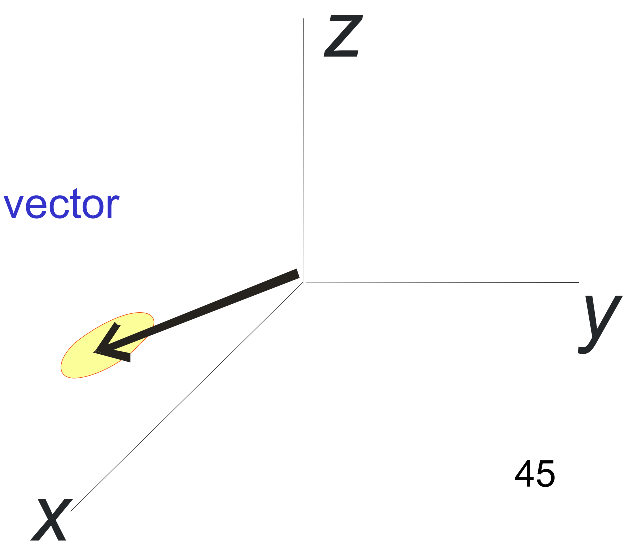
- a collection of two-level atoms can be described in terms of a collective "angular momentum"

$$\vec{S}^a = \sum_{\mu=1}^{N} \frac{1}{2} \vec{\sigma}^{(\mu)}$$

collective angular momentum

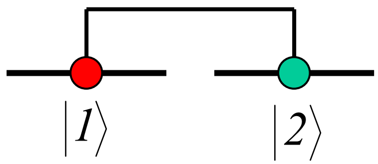
two-level atom = spin -1/2

Bloch vector



atoms cont.

- superposition of the two ground states: coherent spin state



$$\left[\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\right]^{\otimes N}$$

Bloch vector

$$\langle \vec{S}^a \rangle = (\langle S_x^a \rangle, \langle S_y^a \rangle, \langle S_z^a \rangle) = (\frac{N_a}{2}, 0, 0)$$

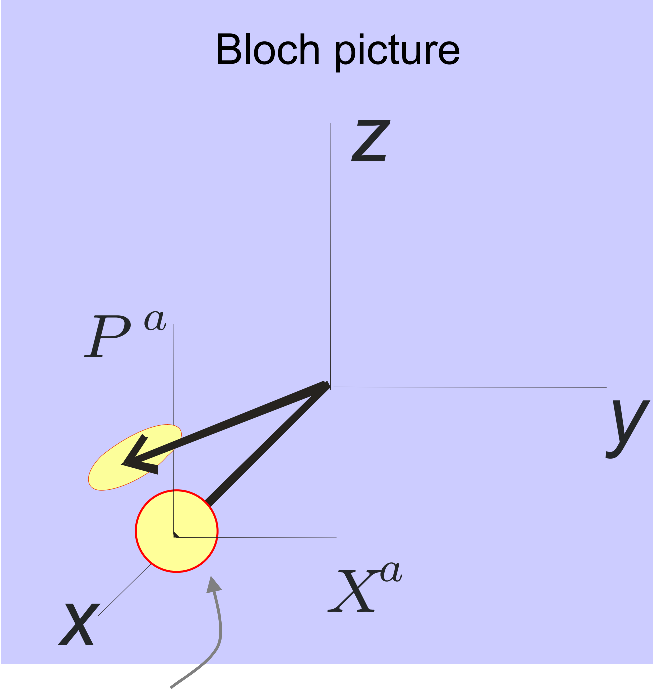
- quantum fluctuations

$$[S_y^a, S_z^a] = iS_x^a \qquad \Delta S_y^a \Delta S_z^a \geq \frac{1}{2}|\langle S_x^a \rangle|$$

we treat S_x^a classically and rescale

$$[X^a, P^a] = i \qquad \Delta X^a \Delta P^a \geq \frac{1}{2}$$

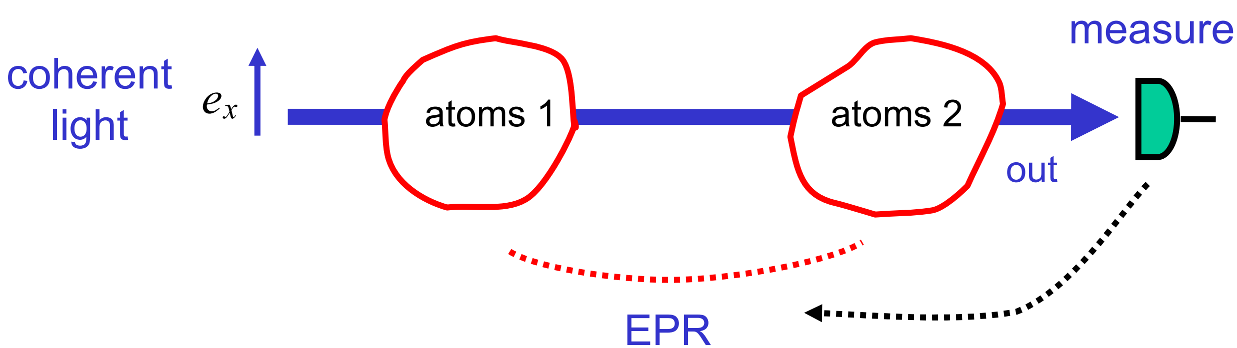
canonical communation relations



- ✓ coherent spin state =vacuum state
- ✓ there are *many* cv quantum states around it:

$$|\psi^a\rangle = \int dX^a |X^a\rangle \psi(X^a)$$

Teleportation with coherent light + atomic ensembles



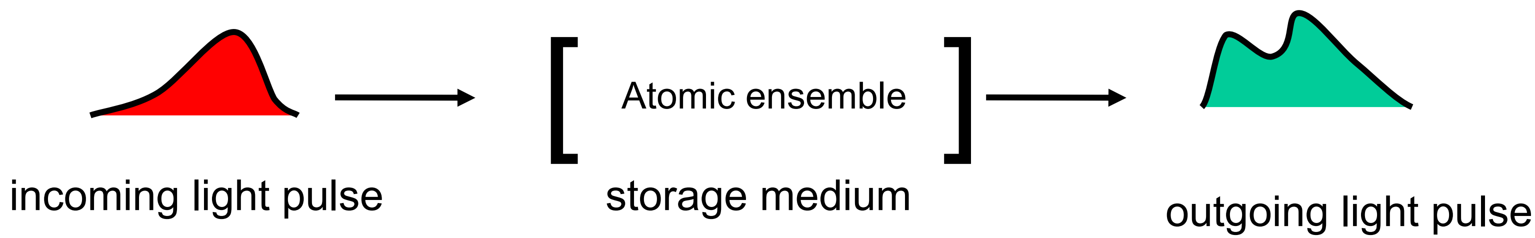
measurement projects atomic ensembles
into continuous variable EPR state

$$|\mathrm{EPR}\rangle \sim \int dP\,|P\rangle_A|-P\rangle_B = \int dX\,|X\rangle_A|X\rangle_B$$

- ✓ theory: Innsbruck
- ✓ experiment: E. Polzik et al. (Aarhus), Nature 2001

Atomic ensembles: quantum memory for light

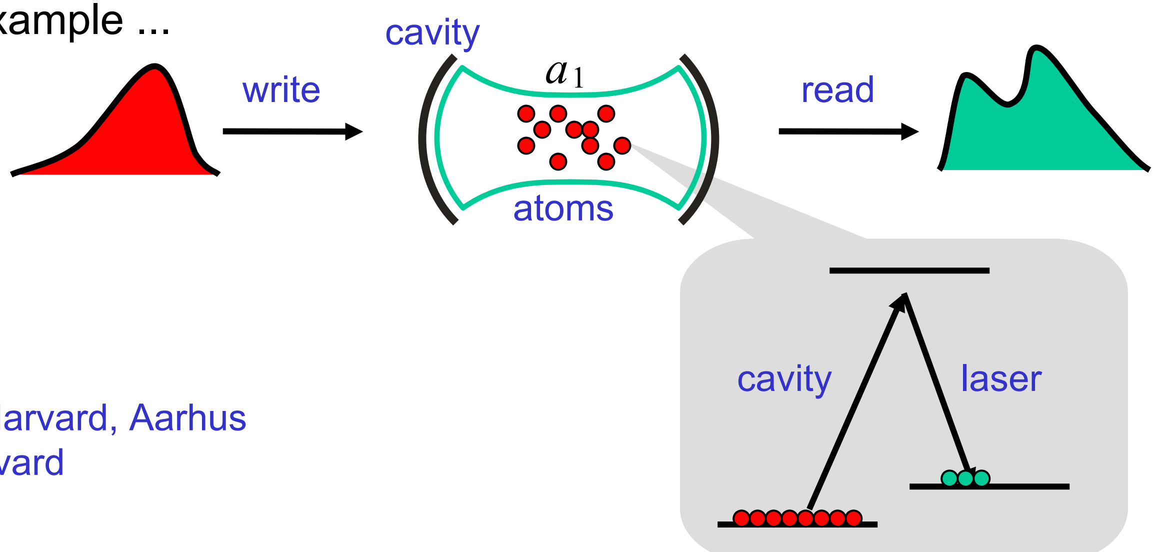
- purpose



✓unknown (arbitrary) state
✓known shape of wave packet

✓same state
✓reshaping

- how? example ...



theory Harvard, Aarhus
exp: Harvard

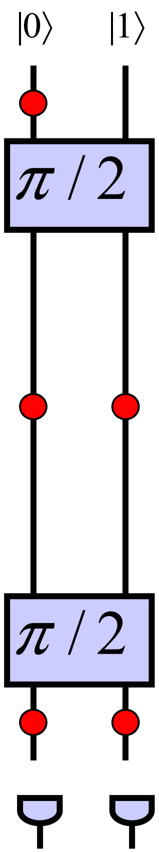
4. Precision measurements

- maximally entangled states and squeezed atomic states for precision measurement beyond the standard quantum limit

theory: NIST Boulder, Innsbruck, Aarhus, Georgia Tech, Harvard

exp: NIST Boulder, Aarhus, Rochester

Ramsey method



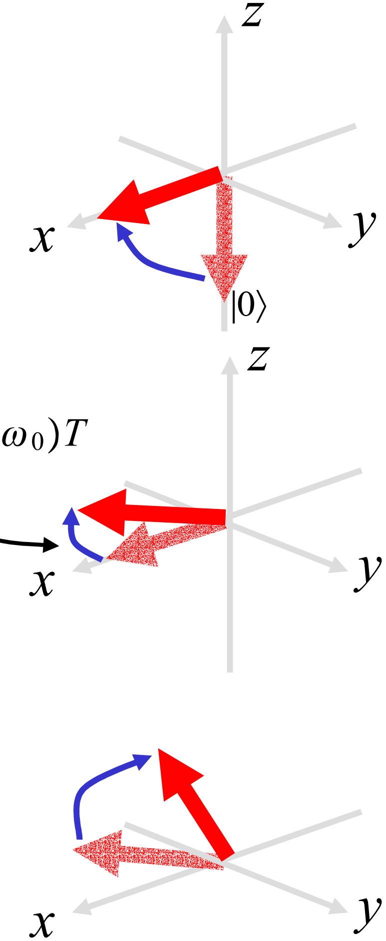
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\rightarrow \frac{1}{\sqrt{2}}(|0\rangle + e^{-i(\omega_L-\omega_0)t}|1\rangle)$$

$$\cos(\tfrac{1}{2}(\omega_L - \omega_0)t)|0\rangle + \sin(\tfrac{1}{2}(\omega_L - \omega_0)t)$$

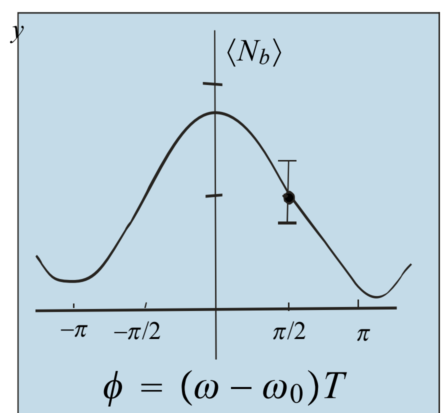
Bloch picture

$\phi = (\omega_L - \omega_0)T$



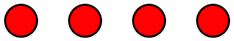
# of atoms in $|0\rangle$

$$P_0(T) = \cos(\tfrac{1}{2}(\omega_L - \omega_0)T)^2$$



We compare ...

- N independent atoms



$$\Delta\omega_{\text{SQL}} = \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{\sqrt{N}}$$

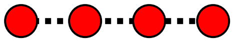
standard quantum noise limit

Remarks:

✓ N is limited: density / collisions

✓ T is limited: decoherence

- N entangled atoms



$$\Delta\omega_{\text{ent}} = \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{f(N)} \geq \frac{1}{T\sqrt{n_{\text{rep}}}}\,\frac{1}{N}$$

Heisenberg limit:
maximally entangled state

$$|0000\rangle + |1111\rangle$$

- figure of merit: to achieve the same uncertainty ...

$$\xi^2 = \frac{(n_{\text{rep}})_{\text{SQL}}}{(n_{\text{rep}})_{\text{ent}}} = \frac{N}{f(N)^2} \geq \frac{1}{N}$$

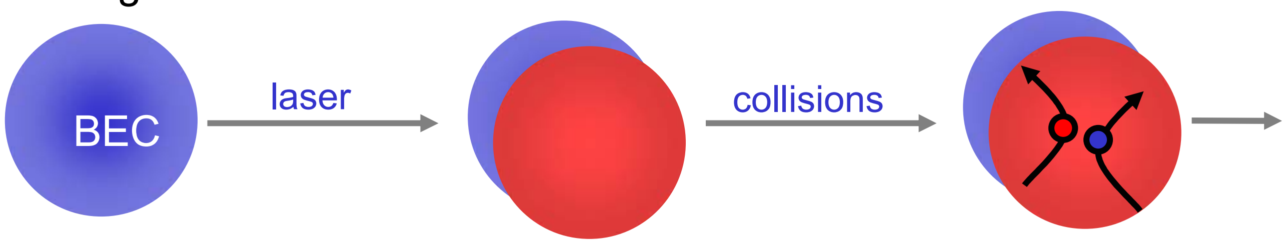
We want $\xi^2 \ll 1$

- present experiments: NIST Boulder 4 ions entangled

theory: Aarhus + Innsbruck, Georgia Tech, Harvard; exp.: see also Yale

Example: Spin Squeezing with BEC

- Entanglement via collisions

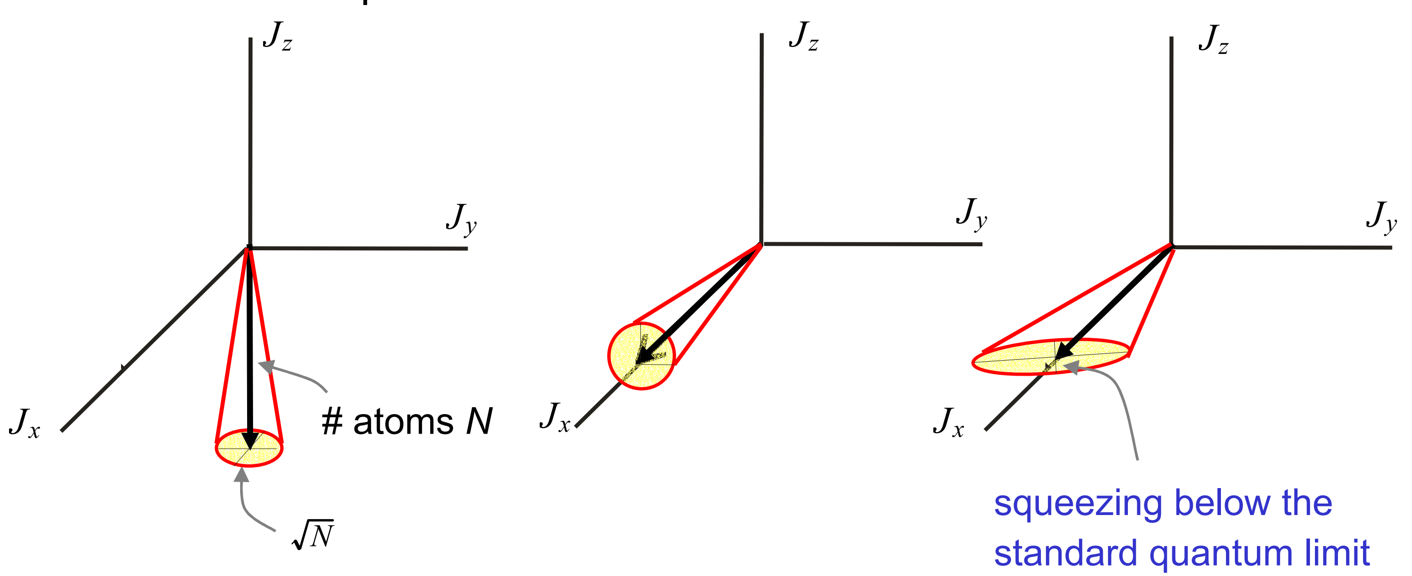


$|0\rangle^{\otimes N}$ product state

$[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)]^{\otimes N}$ product state

$\sum c_n |0\rangle^n |1\rangle^{N-n}$ entangled state

- Bloch vector picture



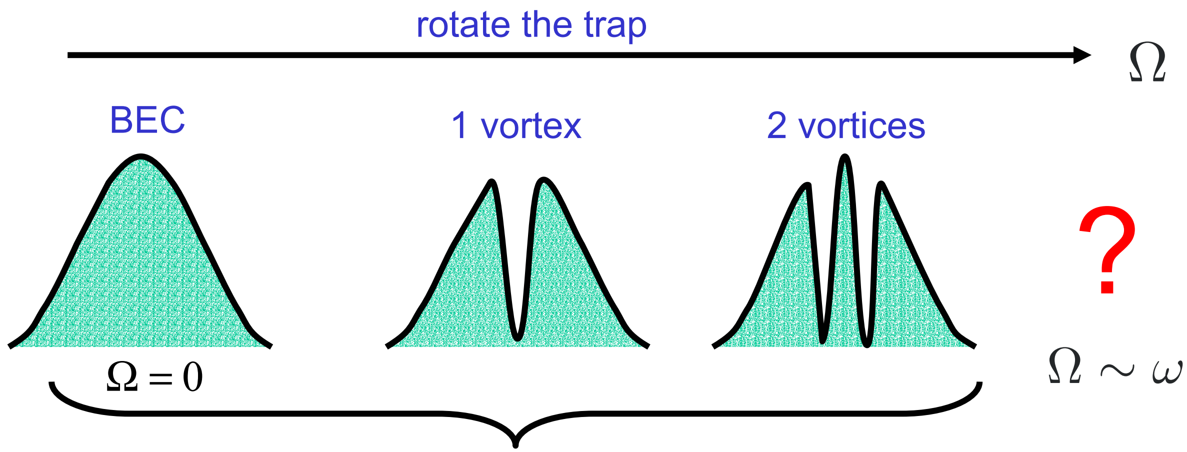
5. Topological quantum computing

concept: Kitaev, Preskill, Freedman

- engineering model systems with *topologically protected ground states*: indistinguishable under local operations
- Fractional quantum Hall effect: electron in 2D + magnetic field: isolated ground state with gapped excitations
- on a closed surface of genus g: renders the *ground state degenerate*.
- transformation of degenerate ground states into each other via motion of flux quantum around nontrivial topological paths
- Rem.: so far no atomic physics models

A (small) first step: ½ - (abelian) anyons in small BECs

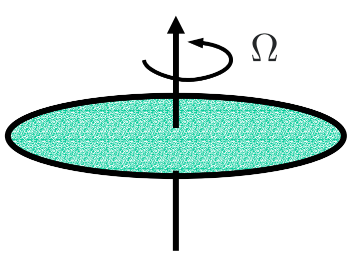
- quantum degenerate Bose gas: 2D and rotating trap



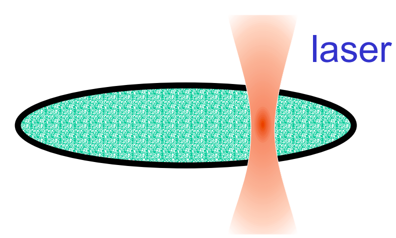
product states

entangled states ☺

- what happens is a fractional quantum Hall scenario with bosons



Laughlin fluid

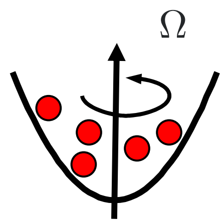


excitations:
½ anyons

we can create and
manipulate anyons!

1. System: N bosonic atoms

- N bosons



✓ harmonic trapping potential ω

✓ rotation Ω

- Hamiltonian in rotating frame

$$H = \frac{1}{2}\sum_{i=1}^{N}\left(-\nabla_i^2 + r_i^2 - 2\frac{\Omega}{\omega}L_{iz}\right) + \eta\sum_{i<j}^{N}\delta(\mathbf{r_i} - \mathbf{r_j})$$

length scale: motion in trap rotation contact interaction

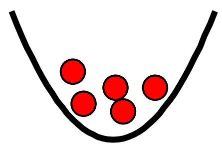
$\ell = (\hbar/m\omega)^{1/2}$

- In the limit $\Omega \sim \omega$ equivalent to (fermionic) quantum Hall Hamiltonian

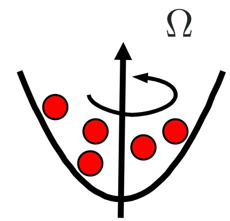
Gunn & Wilkin '00

2. Energy spectrum

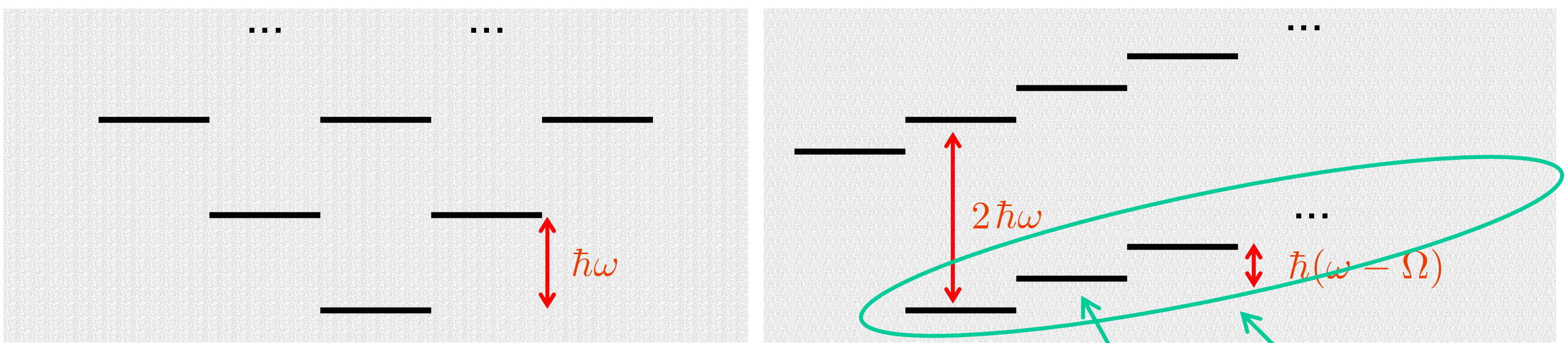
- independent atoms



harmonic trap



+ rotation



Landau wave functions:

$$z^m e^{-\frac{1}{2}|z|^2}$$

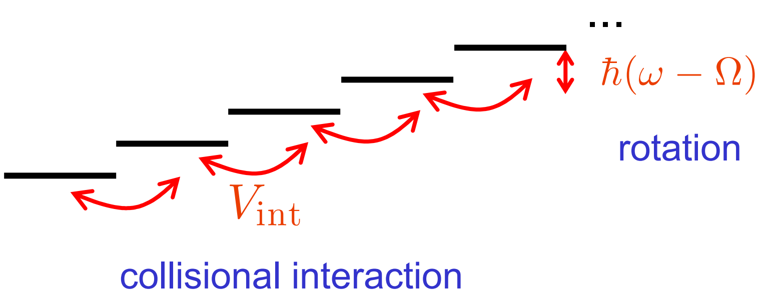
$x + iy$

"Landau levels":

~ degenerate

- we assume: dynamics restricted to first Landau levels $kT \ll \hbar\omega$

- interacting atoms
  - dynamics in first "Landau level"



- for $\Omega \sim \omega$ a parameter regime of *effectively strong* interactions

$$V_{\rm int} \gg \hbar(\Omega - \omega)$$

external parameter

✓ strongly correlated qu fluid

✓ dilute gas!

cont.

- ground state: ½ - Laughlin state

physics: atoms want to be in a *collisional dark state* to minimize energy:

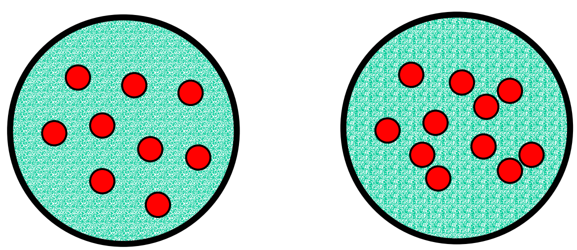
$$V_{\text{int}}\psi = 0$$

$$\psi[z] = \prod_{i<j}(z_i - z_j)^2 \prod_k e^{-|z_k|^2/2}$$

built from lowest Landau levels

avoid interaction

bosons



Laughlin

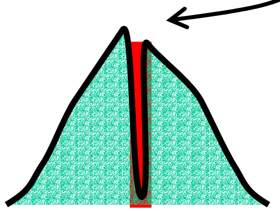
independent atoms

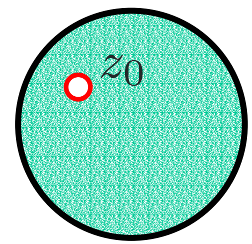
3. Excitations: ½ anyons

- quasihole at position z_0

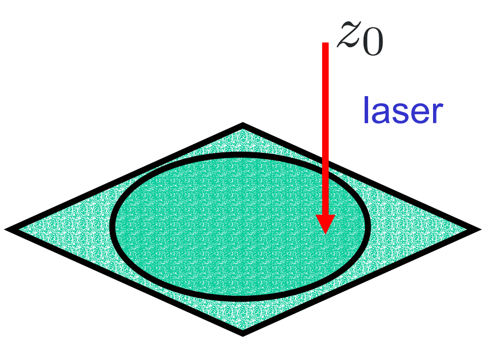
$$\psi_{z_0}[z] = \prod_i (z_i - z_0)\ \psi[z]$$

half atom missing:
½ anyons





- creating a quasiparticle: piercing with am offresonant laser



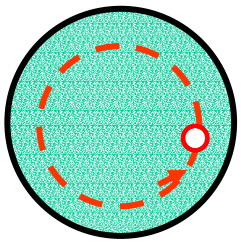
$$H_\circ = H_L + V_0 \sum_i \delta(z_i - z_0)$$

off-resonant laser interaction

4. Anyons & fractional statistics

We adiabatically drag the quasihole around ... and pick up a Berry's phase

- one anyon



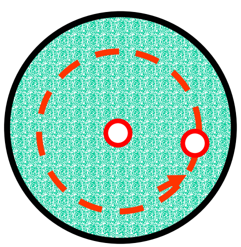
$$\psi_{z_0} \longrightarrow e^{i\phi}\psi_{z_0}$$

$$\phi = 2\pi n$$

Berry's phase

atoms inside the curve

- two anyons



$$\psi_{z_0 z_1} \longrightarrow e^{i\phi}\psi_{z_0 z_1}$$

$$\phi = 2\pi(n - \frac{1}{2}) = 2\pi n - \pi$$

✓ there is an extra π phase

✓ independent of area

✓ if we interchange the two holes: $\pi/2$

½ anyons: $\pi/2$

(fractional statistics)

bosons: 2π

fermions π

Fractional statistics has never been seen directly!